

Supertubes versus superconducting tubes

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Abstract

In this paper we show the relationship between cylindrical D2-branes and cylindrical superconducting membranes described by a generic effective action at the bosonic level. In the first case the extended objects considered, arose as blown up type IIA superstrings to D2-branes, named supertubes. In the second one, the cosmological objects arose from some sort of field theories. The Dirac-Born-Infeld action describing supertubes is shown to be equivalent to the generic effective action describing superconducting membranes via a special transformation.

(Dedicated to Prof. Alberto García on the occasion of his 60th birthday)

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1 Introduction

Nowadays, a number of outstanding problems in physics are currently being worked out using techniques involving the string theory machinery. Every time new ideas transform our understanding of string theory. Since their appearance, the so-called Dp -branes [1, 2] paved the way for a fierce study in string theory because it is assumed that they are source for the Ramond-Ramond fields in Type II theories besides of being a useful tool to explain several supersymmetric and non-supersymmetric field theories. Among main features of Dp -branes, resides that 10-dimensional superstrings can end on them. The incorporation of Dp -branes in superstring picture gives a lot of issues in the theory of solitonic states in nonperturbative string theory and permits to reveal different aspects of string/ M -theory dualities.

Recently, special $D2$ -branes named supertubes, have emerged as worldvolume realizations of some kind of sigma models [3, 4], exhibiting more features for the intriguing M -theory. The role played by Born-Infeld fields describing cylindrical $D2$ -branes is to maintain stability against tension, including supergravity forces. On the other hand, at cosmological context, some field theories predict the existence of topological defects like strings or domain walls holding ability of carry some kind of charge [5, 6, 7, 8, 9, 10]. For these superconducting strings/membranes, a scalar field living on the worldsheet plays a similar role as before, i.e., maintain stability against tension. Due to previous similarities, it is to be expected similar descriptions in their dynamics, i.e., must exist a transformation between both dynamical descriptions.

In this paper we shall concentrate on the dynamical similarities between supertubes and superconducting cylindrical branes which are extended objects arose at first instants of the universe. The paper is organized as follows. In Sect. 2 we explain the notation we use through the paper and obtain the canonical momenta associated to the embedding variables by means of variational techniques and with the help of Noether theorem [11, 12], in order to identify the energy density. In Sect. 3 we describe the main results about supertubes developed in [3]. In Sect. 4 we write the generic effective action which provides us the dynamics of superconducting extended objects. The comparison of both dynamics is given in Sect. 5. Finally we end the paper with some comments.

2 Noether currents and momenta

We consider a Dp -brane of dimension d evolving in a $N + 1$ dimensional background spacetime with metric $g_{\mu\nu}$, $\mu, \nu = 0, 1, \dots, N$. The Dp -brane worldvolume is an oriented

timelike manifold of dimension d , usually denoted by m , endowed with an induced metric γ_{ab} from the bulk. If $x^\mu = X^\mu(\xi^a)$ are the embedding functions of the Dp -brane, the induced metric on the worldvolume is given by $\gamma_{ab} = g_{\mu\nu} X^\mu_{,a} X^\nu_{,b}$. ξ^a denotes the worldvolume coordinates.

We demand that the Dirac-Born-Infeld action (DBI) specify the motion of the Dp -brane,

$$S_{BI} = \alpha \int_m d^{d+1} \xi \sqrt{-\det(\gamma_{ab} + F_{ab})}, \quad (2.1)$$

where α is the tension of the Dp -brane, $F_{ab} = 2\partial_{[a}A_{b]}$ is the electromagnetic tensor associated to the worldvolume $U(1)$ gauge field A_a ; $a, b = 0, 1, \dots, d$.

A general variation of the action (2.1) always cast out in the form [11, 12]

$$\begin{aligned} \delta S_{BI} &= \int_m \left\{ \mathcal{E}_\mu \delta X^\mu + \mathcal{E}^a \delta A_a + \sqrt{-\gamma} \nabla_a F^a [\delta X; \delta A] \right\}, \\ &= \int_m \left\{ \mathcal{E}_\mu \delta X^\mu + \mathcal{E}^a \delta A_a \right\} + \int_{\partial m} \left\{ P_\mu \delta X^\mu + \pi^a \delta A_a \right\}, \end{aligned} \quad (2.2)$$

where \mathcal{E}_μ and \mathcal{E}^a are the Euler Lagrange derivatives of \mathcal{L}_{BI} with respect to X^μ and A_a , respectively; F^a is an operator defined on the worldvolume and it is related to the momenta associated to the configuration space. The argument of the F^a , is indicated within the square bracket. P_μ and π^a are canonical momenta associated to the configuration space. We are interested in the momentum P_μ given by

$$P_\mu = \sqrt{h} T^{ab} \eta_a X_{\mu,b}, \quad (2.3)$$

where T^{ab} is the energy-momentum tensor (see (2.7)) and η_a is the unit timelike normal vector on the boundary ∂m . In fact, when one perform a foliation of the worldvolume in spacelike hypersurfaces Σ , the corresponding normal vector is η^a .

When the classical equations of motion are satisfied, $\mathcal{E}_\mu = 0$ and $\mathcal{E}^a = 0$, these equations are equivalent to the set

$$T^{ab} K_{ab}^i = 0, \quad (2.4)$$

$$\nabla_a T^{ab} = 0, \quad (2.5)$$

$$\nabla_a \mathcal{J}^{ab} = 0, \quad (2.6)$$

where T^{ab} denotes the symmetric energy-momentum tensor, \mathcal{J}^{ab} denotes an antisymmetric bicurrent density and K_{ab}^i denotes the extrinsic curvature of the worldvolume, where i runs from 1 to $N + 1 - d$, *i.e.*, i labels the number of unit normals to the worldvolume. Explicitly, T^{ab} and \mathcal{J}^{ab} are given by

$$T^{ab} = \frac{\sqrt{-M}}{\sqrt{-\gamma}} (M^{-1})^{(ab)}, \quad (2.7)$$

$$\mathcal{J}^{ab} = \alpha \sqrt{-M} (M^{-1})^{[ab]}, \quad (2.8)$$

where $(M^{-1})^{ab}$ means the inverse matrix of $M_{ab} := \gamma_{ab} + F_{ab}$ and $M = \det(M_{ab})$.

In order to illustrate the previous results, from (2.1), the corresponding Lagrangian density for $D2$ -branes is

$$\mathcal{L} = \sqrt{-\gamma} \left(1 + \frac{1}{2} F_{ab} F^{ab} \right)^{1/2}, \quad (2.9)$$

where γ denotes the determinant of γ_{ab} . In such case, we are able to write explicitly both the symmetric and antisymmetric parts of the inverse matrix of M_{ab} , namely

$$(M^{-1})^{(ab)} = \gamma^{ab} - (1 + F^2)^{-1/2} F^{c(a} F_c^{b)}, \quad (2.10)$$

$$(M^{-1})^{[ab]} = - (1 + F^2)^{-1/2} F^{ab}, \quad (2.11)$$

where we use the notation $F^2 = (1/2) F_{ab} F^{ab}$.

3 Supertubes

Brane expansion is an interesting effect in physics. A current topic in Dp -brane physics concerns expansions from one configuration to another. Interaction of Dp -branes with external fields (not necessarily at supersymmetric level), under certain circumstances, produce expansions in order to stabilize a given configuration. It has been recently shown [3] that type IIA strings can blow up to produce cylindrical $D2$ -branes, being the angular momentum the stabilizer for maintaining a finite radius for the cylinder.

Following [3], consider a static tubular $D2$ -brane with radius $R(z, \phi)$, embedded in a $N = 10$ Minkowski spacetime

$$ds^2 = -dT^2 + dZ^2 + dR^2 + R^2 d\Phi^2 + ds^2(E^6), \quad (3.12)$$

with worldvolumen coordinates $T = t, Z = z, \Phi = \phi$, where the axis is along the z direction. The induced metric is given by

$$ds^2 = -dt^2 + dz^2 + R^2 d\phi^2 + (\partial_z R dz + \partial_\phi R d\phi)^2. \quad (3.13)$$

Assuming time independent Born-Infeld fields via the 2-form field strength

$$F = E dt \wedge dz + B dz \wedge d\phi, \quad (3.14)$$

the $D2$ -brane Born-Infeld Lagrangian (2.1) is

$$\mathcal{L} = \sqrt{(R^2 + R_\phi^2)(1 - E^2) + B^2 + R^2 R_z^2}, \quad (3.15)$$

where $R_\phi = \partial_\phi R$ and $R_z = \partial_z R$. Defining the electric displacement $\Pi \equiv \partial \mathcal{L} / \partial E$, the associated Hamiltonian density to (3.15) results

$$\mathcal{H} = \Pi E - \mathcal{L}. \quad (3.16)$$

There is a relation between the electric field and the electric displacement given by

$$E = \frac{\Pi}{R} \sqrt{\frac{B^2 + R^2}{\Pi^2 + R^2}}, \quad (3.17)$$

such in the case when R is constant and the energy density becomes

$$\mathcal{E} = \frac{1}{R} \sqrt{(\Pi^2 + R^2)(B^2 + R^2)}. \quad (3.18)$$

The z independence of the electric displacement comes from the fact that Π is subject to the Gauss law constraint.

4 Superconducting tubes

To describe superconducting branes we consider the effective action [5, 6, 7, 8, 9, 10]

$$S_c = \int_m d^{d+1} \xi \sqrt{-\gamma} L(\omega), \quad (4.19)$$

where $L(\omega)$ denotes a specific model depending on internal fields acting on the brane through the combination $\omega := \gamma^{ab} \phi_{,a} \phi_{,b}$. There is an important model developed by Nielsen [13], which consists basically in consider a scalar field living on the worldvolume which, under certain circumstances, prevents the collapse. For the case of a superconducting membrane, this model is given by

$$L = \sqrt{k_1 + k_2 \omega}, \quad (4.20)$$

where k_1 and k_2 are constants. The corresponding equations of motion are (compare with (2.4) and (2.6))

$$T^{ab} K_{ab}^i = 0, \quad (4.21)$$

$$\nabla_a J^a = 0, \quad (4.22)$$

where the explicit form of the energy-momentum tensor is

$$T^{ab} = L(\omega) \gamma^{ab} - 2 \frac{dL}{d\omega} \gamma^{ac} \gamma^{bd} \phi_{,c} \phi_{,d}. \quad (4.23)$$

Note that the second equation (4.22) corresponds to a conservation law being the conserved current defined as $J^a = 2 \frac{dL}{d\omega} \gamma^{ab} \nabla_b \phi$.

We assume a background 4-dimensional Minkowski metric of the form

$$ds^2 = -dt^2 + dr^2 + r^2 d\theta^2 + dz^2, \quad (4.24)$$

and we specialize to a superconducting tube described by the embedding

$$x^\mu(t, \theta, z) = \begin{pmatrix} t \\ r(t) \\ \theta \\ z \end{pmatrix}, \quad (4.25)$$

where the radius $r = r(t)$ is θ and z independent. The induced metric on the tube is given by

$$ds^2 = (-1 + \dot{r}^2) dt^2 + r^2 d\theta^2 + dz^2. \quad (4.26)$$

Under the following assumption for the scalar field, $\phi = \phi_1(t) + N\theta$, and from the conservation Eq. (4.22) we get how evolve in time the scalar field

$$\dot{\phi} = -\frac{\Omega\sqrt{1-\dot{r}^2}}{2r(dL/d\omega)}, \quad (4.27)$$

where Ω is an integration constant and N is a constant to be determined. It is straightforward to obtain the form of ω

$$\omega = \frac{N^2 - (\Omega^2 k_1/k_2)^2}{r^2 + (\Omega^2/k_2)}. \quad (4.28)$$

Handling the previous results allow us to write the explicit dependence of the Lagrangian (4.20),

$$L = \sqrt{\frac{k_1 r^2 + k_2 N^2}{r^2 + (\Omega^2/k_2)}}. \quad (4.29)$$

5 Mechanical equivalence between tubes

Now we turn to show the similarity in dynamics between supertubes and superconducting tubes. When R is θ and ϕ independent, this fact allows us to rewrite the Lagrangian as

$$\mathcal{L} = -\sqrt{R^2(1 - E^2) + B^2}. \quad (5.30)$$

The constant N appearing in the definition of the scalar field, can be computed in terms of Π using the relation between (4.22) and (2.5). Inserting the results for $L(\omega)$ and $dL/d\omega$ in the definition of the electric displacement, we get $\Pi = (k_2/k_1)^{1/2} N$.

We look at the zero-zero component in the energy-momentum tensor

$$\begin{aligned} T^{00} &= L(\omega)\gamma^{00} - 2\frac{dL}{d\omega}(\gamma^{00})^2\dot{\phi}^2 \\ &= \frac{1}{r^2(-1+\dot{r}^2)}\sqrt{(k_1r^2+k_2N^2)(k_2r^2+\Omega^2)} \end{aligned} \quad (5.31)$$

such that, for the static configuration $r = R$ the energy density can be extracted from the time component of the momentum vector (2.3), i.e.,

$$\begin{aligned} \mathcal{E} &= \frac{1}{R}\sqrt{(k_1R^2+k_2N^2)(k_2R^2+\Omega^2)}, \\ &= \frac{1}{R}\sqrt{k_1(R^2+\pi^2)(k_2R^2+B^2)}. \end{aligned} \quad (5.32)$$

If we choose $B = \Omega$ and $k_1 = k_2 = 1$ then $\Pi = N$ and we get the energy in accordance with the result arose from Born-Infeld action (3.18), specifically for the case of supertubes [3]. It is well known [5, 8, 10] that equilibrium configurations for superconducting extended objects are reached thanks to the current produced by the scalar field, avoiding the collapse. Actually, ϕ plays a similar role as the angular momentum for the supertubes case.

6 Concluding remarks

We have shown an equivalence, at the bosonic level, between supertubes and superconducting tubes. Superconducting membranes are described uniquely by a scalar field living on its worldvolume. The energy in both tube cases turns out to be same if we relate the conserved type IIA string charge and the D0-brane charge per unit length carried by the tube with the current and the charge of the superconducting tube, respectively. The next task is to explore the possibility to introduce supersymmetry in the superconducting membranes approach and show explicitly its character 1/4 supersymmetric in order to complete the proof of total equivalence between supertubes and superconducting tubes. The last issue is under current investigation.

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